

25UMBT401	PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS	Category	L	T	P	R	Credit	Year of Introduction
		BSC	3	0	0	0	3	2025

Preamble:

This course introduces students to the modern theory of probability and random processes, covering important models of random variables and analysis of random processes. A brief course in numerical techniques familiarizes students with some basic numerical techniques which are especially useful when analytical solutions are hard to find.

Prerequisite: Nil

Course Outcomes: After the completion of the course, the student will be able to:

CO	Course Outcomes	Knowledge Level
CO1	Explain the concept, properties and important models of discrete random variables and to apply in suitable random phenomena.	K3 Apply
CO2	Explain the concept, properties and important models of continuous random variables and to apply in suitable random phenomena.	K3 Apply
CO3	Interpret random processes using autocorrelation, power spectrum and Poisson process model as appropriate.	K3 Apply
CO4	Apply numerical methods to find solutions of linear system of Equations, ordinary differential equations and Laplace equations.	K3 Apply

Mapping of course outcomes with program outcomes and program specific outcomes

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11
CO1	3	2	2	2	-	-	-	-	-	-	1
CO2	3	2	2	2	-	-	-	-	-	-	1
CO3	3	2	2	2	-	-	-	-	-	-	1
CO4	3	2	2	2	-	-	-	-	-	-	1
Avg	3	2	2	2		-	-	-	-	-	1

Mapping of course outcomes with Knowledge and Attitude Profile (WK)

	WK1	WK2	WK3	WK4	WK5	WK6	WK7	WK8	WK9
CO1	*	*	*	*	*			*	
CO2	*	*	*	*	*			*	
CO3	*	*	*	*	*			*	
CO4	*	*	*	*	*			*	

Wks mapped with the course

WK1	WK2	WK3	WK4	WK5	WK6	WK7	WK8	WK9
*	*	*	*	*			*	

Mark Distribution:

Total Marks	CIE Marks	ESE Marks	ESE Duration
100	40	60	2.5 hrs

Continuous Internal Evaluation Pattern:

Attendance	: 5 marks
Continuous Assessment Test	: 20 marks
Continuous Assessment Assignment	: 15 marks

SYLLABUS

Module 1 (11hrs)

Random variables, discrete random variables and their probability distributions, Cumulative distribution function, Expectation, Mean and variance, Binomial distribution, Poisson distribution, Poisson distribution as a limit of the binomial distribution, Joint pmf of two discrete random variables, Marginal pmf, Independent random variables, Expected value of a function of two discrete variables.

Module 2 (11hrs)

Continuous random variables and their probability distributions, Cumulative distribution function, Expectation, Mean and variance, Uniform, Normal and Exponential distributions, Joint pdf of two Continuous random variables, Marginal pdf, Independent random variables, Expectation value of a function of two continuous variables.

Module 3 (10hrs)

Random processes and classification, mean and autocorrelation, wide sense stationary (WSS) processes, autocorrelation and power spectral density of WSS processes and their properties, Poisson process-distribution of inter-arrival times, combination of independent Poisson processes(merging) and subdivision (splitting) of Poisson processes (results without proof).

Module 4 (10hrs)

Newton-Raphson Method, Gauss Elimination Method, Gauss – Jordan Method, Numerical solution of ordinary differential equations-Euler's method, Modified Euler's method, Runge - Kutta method of 2nd Order, Numerical solution of Laplace equation –Jacobi's Method, Curve Fitting by Method of Least Squares - Straight lines, Parabola.

Textbooks

- 1.Devore, J. L. (2016). *Probability and statistics for engineering and the sciences* (9th ed.). Cengage Learning.
- 2.Ibe, O. C. (2005). *Fundamentals of applied probability and random processes*. Elsevier.
- 3.Sastry, S. S. (2012). *Introductory methods of numerical analysis* (5th ed.). PHI Learning Pvt. Limited.

Reference Books

1. Papoulis, A., & Pillai, S. U. (2002). *Probability, random variables and stochastic processes* (4th ed.). McGraw-Hill.
2. Palaniammal, S. (n.d.). *Probability and random processes* (3rd ed.). PHI Learning Private Limited.
3. Chapra, S. C., & Canale, R. P. (2021). *Numerical methods for engineers* (8th ed.). McGraw-Hill Education.

END SEMESTER EXAMINATION–MODEL QUESTION PAPER

CourseCode	25UMBT401
Course Name	PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS
Max.Marks:60	Duration:2.5Hours

Part A <i>(Answer all questions. Each question carries 3 marks)</i>					
QnNo.	Question	Marks	CO	BL	
1	Determine the Binomial distribution for which mean is 5 and variance $\frac{15}{4}$	3	1	L2	
2	Find the probability distribution function, if the Random variable X takes the values 1, 2, 3, 4 such that $2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4)$	3	1	L3	
3	Find the value of k and Mean for the p d f $f(x) = \begin{cases} kx^2, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$	3	2	L2	
4	A Random variable X has a Uniform distribution in $(-4, 4)$. Find $(X \leq 2)$.	3	2	L3	
5	State any three properties of autocorrelation function.	3	3	L2	
6	Find the power spectral density of the WSS power process with autocorrelation function $2e^{-\frac{1}{2}\tau} \cos \tau + 4e^{-\frac{1}{2}\tau} \sin \tau$	3	3	L3	
7	Find the positive root of the equation $2x^3 - 3x - 6 = 0$ correct to 4 decimal places by Newton - Raphson method.	3	4	L2	
8	Use Euler method to compute $y(0.15)$ for $y' = x + y + xy$, $y(0) = 1$ by taking $h = 0.15$	3	4	L2	
Part B <i>(Answer any one full question from each Module. Each carry 9 marks)</i>					
Module I					
Qn No.	Question	Marks	CO	BL	
9a	A random variable X has the following PMF	5	1	L2	
	<table border="1"> <tr> <td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr> </table>				x
x	0	1	2	3	

	$f(x)$ $\frac{k}{2}$ $\frac{k}{3}$ $\frac{k+1}{3}$ $\frac{2k-1}{6}$			
	Find (i) the value of k (ii) mean and variance of X			
b	In a certain industrial facility, it is known that the probability of an accident on any given day is 0.005 and accidents are independent of each other. Use Poisson distribution to find the probability that in any given period of 400 days (i) there will be an accident on one day (ii) there are at most three days with an accident?	4	1	L3
OR				
10a	Derive mean and variance of Binomial distribution	5	1	L2
b	The joint distribution of (X,Y) is given by $(x, y) = (x + y); x = 0, 1, 2; y = 0, 1, 2, 3.$ Find (i) the value of k (ii) the marginal distribution.	4	1	L3
Module II				
Qn No.	Question	Marks	CO	BL
11a	A random variable X has probability density function $f(x) = \begin{cases} \frac{3}{4}(1-x^2), & -1 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$ Find the mean and variance of $Y = 2X - 3$	5	2	L2
b	The time in hours required to repair a machine is exponentially distributed with mean 20. What is the probability that the required time (i) exceeds 30 hours? (ii) between 16 hours and 24 hours?	4	2	L3
OR				
12a	Suppose the diameter at breast height of trees of a certain type is normally distributed with mean 8.8 and standard deviation 2.8 (i) What is the probability that the diameter of a randomly selected tree will be at least 10 inches? (ii) What is the probability that the diameter of a randomly selected tree will exceed 20 inches? (iii) What is the probability that the diameter of a randomly selected tree will be between 5 inches and 10 inches?	4	2	L3
b	The joint PDF of continuous random variable X and Y is given by $f(x,y) = \begin{cases} kxy, & 0 < x < 4, 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$ Find (i) the value of k (ii) marginal distributions (iii) Check whether X and Y are independent.	5	2	L2
Module III				
Qn No.	Question	Marks	CO	BL
13. a	A random process $X(t)$ is defined by $X(t) = 3 \cos(4t + \theta)$, where θ is uniformly distributed in $[0, 2\pi]$. Find the mean, auto correlation and auto covariance.	5	3	L3

13. b	Find the power spectral density function of the WSS process whose autocorrelation function is given by $R_X(\tau)=\frac{a^2}{\tau} \cos (\omega \tau)$	4	3	L2												
OR																
14. a	If X(t) is a WSS process with autocorrelation $R_X(\tau)=\frac{\tau^2+44}{\tau^2+4}$. Find the mean and variance.	5	3	L3												
14. b	Prove that sum of two independent Poisson processes is a Poisson process.	4	3	L2												
Module IV																
Qn No.	Question	Marks	CO	BL												
15. a	Use Gauss – Jordan method to solve the system $4x + 3y - z = 6$ $3x + 5y + 3z = 4$ $x + y + z = 1$	4	4	L3												
15. b	Solve Laplace’s equation $\nabla^2 u=0$ on $0\leq x,y\leq 1$ with boundary conditions $u(0,y)=0$, $u(1,y)=100$, $u(x,0)=0$ and $u(x,1)=0$. Using a grid with spacing $h=0.25$, write the finite –difference equations and apply the Jacobi’s method for two iterations starting with all interior values equal to zero.	5	4	L3												
OR																
16. a	Use modified Euler’s method to find the value of y at $x=0.25$ and $x=0.5$ given $\frac{dy}{dx}=2xy$, $y(0)=1$	5	4	L3												
16. b	Fit a parabola of the form $y=ax^2+bx+c$ to the following data <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>y</td><td>1.2</td><td>1.7</td><td>2.1</td><td>2.8</td><td>5.9</td></tr></table>	x	0	1	2	3	4	y	1.2	1.7	2.1	2.8	5.9	4	4	L3
x	0	1	2	3	4											
y	1.2	1.7	2.1	2.8	5.9											
